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QUANTIZATION AND SOURCE ENCODING WITH A FIDELITY CRITERION: A S--ETC(U)

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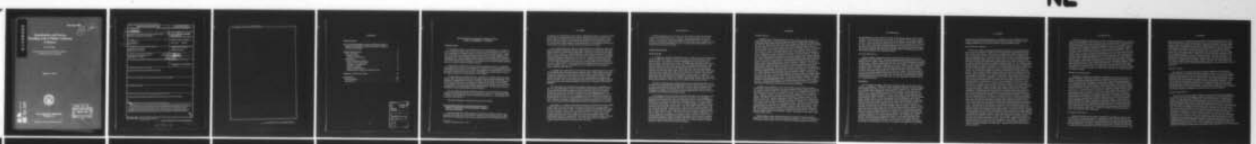

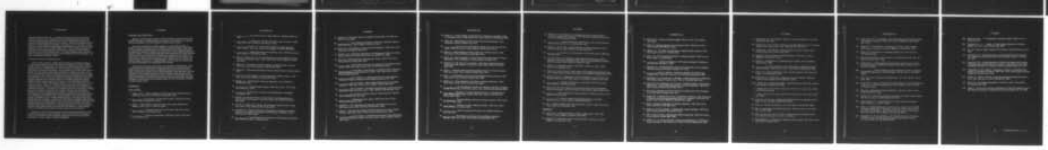

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Quantization and Source Encoding with a Fidelity Criterion: A Survey

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Quantizers have been considered as tools for encoding discrete time information sources for a variety of reasons. Some of these reasons are simple implementation, familiarity to engineers, and an available volume of associated research results. This survey provides a brief analysis of an exhaustive list of references, which has been categorized with respect to significant research and development issues in the areas of quantization, source encoding, and their interrelationship.

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CONTENTS

QUANTIZATION AND SOURCE ENCODING WITH A FIDELITY CRITERION: A SURVEY

INTRODUCTION

In information theory, it is generally known that characterizations of the optimum source encoder and source code for a given source are extremely difficult to obtain. In fact, a large part of information theory is devoted to this and similar problems. Evaluation of the optimum encoding performance is also difficult, but more progress has been made in this area. Such evaluation is usually based on a theoretically derived optimum performance curve, called the rate-distortion function, which is obtainable without knowledge of the optimal encoder description. The solutions to these two companion problems have been the goal of many information theorists.

Optimal source encoding for an entire class of information sources is considerably more difficult and complex. In this situation, the evaluation of the best possible encoding performance has been carried out in only a few instances, and characterization of the optimal source encoder has not been achieved. Implementation of optimal source encoders for classes of sources, it is hoped, will not be too complex.

For these reasons, quantizers have been considered for encoding discrete time information sources and classes of these sources because of their simple implementation, their familiarity to engineers and the volume of associated research results. In this survey we provide a brief analysis of an exhaustive list of references up through 1975, which has been categorized according to the significant research and development issues in the areas of quantization, source encoding, and their interrelationship. The goal is to provide an information base for further research and development.

The next section discusses the relationship between source encoding with a fidelity criterion and quantization; it thus serves as a prelude to the survey. The literature survey is divided into two topic areas, source encoding and quantization. A brief summary and conclusion constitute Sec. 4.

The author apologizes to those whose work has been omitted.

RELATIONSHIP BETWEEN SOURCE ENCODING WITH A FIDELITY CRITERION (RATE-DISTORTION THEORY) AND QUANTIZATION

Source encoding with a fidelity criterion, as introduced by Shannon [52], or rate-distortion theory [6], as it is now called, is a major area of information theory [1-63] and is also the mathematical basis for data compression [6,14]. Rate-distortion theory

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is concerned with specification of the noiseless communication channel or noiseless memory capacity for an acceptable reproduction error or distortion level at the receiver or user. The rate-distortion function is the basis for this specification and the foundation of the theory. Hence, knowledge of the rate-distortion function is of considerable benefit to the communication system designer, and the determination of rate-distortion functions for various sources and classes of sources has been an active area of research.

The rate-distortion function R(d) for a given information or signal source and a given distortion measure, specifies the minimum channel capacity or the minimum source information required for reproducing the entire source output at the receiver with an average distortion of d or less (i.e., to achieve fidelity d). This function is computed by minimizing the mutual information, between the source and the channel output, over all possible channels that generate a distortion level of d or less. In general, this computation uses variational techniques [6,11,33] and is not easy. For a distortion measure that assigns $d = 0$ (for perfect reproduction of the input at the output), R(0) is just the entropy of the source. R(d) decreases monotonically with d and usually becomes zero at some finite value of distortion, say d_{max}, indicating that no information need be transmitted to achieve a distortion value of d_{max}.

An equally active research area is the search for source-encoding techniques to achieve the rate-distortion function values for given distortion levels. Early research in this area dealt almost exclusively with the search for optimal source-encoding techniques yielding performance improvements that would justify their cost and added complexity. This research was fruitful only in some areas, e.g., space communications and high-density data storage [59]. Many source-encoding theorists recently have been engaged in finding good suboptimal source-encoding techniques that are easy to implement [1,2,13,35,40, 56,59,63].

The main objective of using a source-encoding technique such as quantization is to achieve an acceptable balance between the distortion in reproducing the input and the cost of acquiring, transmitting, and/or storing the additional data needed for perfect reproduction of the input. Amplitude quantization of source signals is probably the first source-encoding technique, and it is easy to implement [80,90,93,98,111], e.g., in analog-to-digital conversion. In fact, quantization can be considered the cornerstone of sampled-data theory and systems [68,83,90,98,101,110,112]. Considerable research has been conducted on complete or partial source encoding of stochastic sources by quantization [9,40,64,66,68,70,72,73,75,82,84,87,92,94,97,99,100,102,109,111,112,114,116].

Quantization is a nonlinear, zero-memory operation of transforming a source variable, possibly random and possibly having a continuum of values, into an output variable having a finite number (N) of values. The device performing this transformation is called an N-level quantizer, and its output is an N-step approximation to its input. Distortion results from trying to reproduce the input from the output. A fidelity criterion (for example a particular value of d) is assigned to provide a quantitative measure of the approximation accuracy of reproduction. The most common fidelity criterion is a bound on the mean weighted quantization error, where this error is just the nonnegatively weighted difference between the quantizer input and output.

Optimal quantization of a given source is a scheme that optimizes the distortion measure for that particular source. In general, such quantizers perform poorly with respect to other sources or distortion measures. Robust quantization, of a class of sources, is a quantization scheme that quarantees a performance level or fidelity criterion for all sources in that class [40,95,111,112]. In general, robust quantization is sub-optimal for most sources in the class.

LITERATURE SURVEY

Source Encoding

C. E. Shannon presented the information theory in 1949 [51] and the rate-distortion theory in 1959 [52]. Berger's book [6] is the only one devoted exclusively to rate-distortion theory. He includes an extensive bibliography and covers most previous research results. Other books, such as those of Gallager [21] and Sakrison [49], have at least one chapter covering the basics of this theory. Published papers devoted to rate-distortion theory per se are those of Berger [3,5], Berger and Yu [8], Davisson and Gray [16], Davisson and Pursley [17], Goblick [24], Gray [32], Gray and Davisson [28-30], Gray and Wyner [31], Kieffer [36], Leiner [37], Leiner and Gray [38], Rubin [42], Wolf and Ziv [58], Wyner [62], and Ziv [63, Part II]. Results of the evaluation and/or bounding of rate-distortion (distortion-rate) functions (also the ϵ-entropy of Kolmogorov) were reported by many researchers, including Berger [3], Binia et al. [10], Blahut [12], Dobrushin [19,20], Gray [25-27], Haskell [33], Marton [39], Morris [40], Rubin [42,43], Sakrison [47,48], Tan and Yao [55], and Wyner and Ziv [60].

Although memoryless source models represent many meaningful applications it has generally been acknowledged that substantial benefits from source coding can be achieved for sources with memory [3,4,6,17,25-27,35,53]. Likewise, distortion measures incorporating symbol context dependences [8] or the concept of source coding when side information is available to the encoder-decoder pair [32,37,38,62] have been found to be applications with substantial coding gain potential.

The investigation of source-encoding techniques that achieve the performance promised by rate-distortion theory has been an active area of research. This investigation parallels the earlier research for channel encoding techniques to achieve the performance promised by the channel coding theorems of information theory. In the same manner, this newer area of research is a major interest of coding theorists (e.g., Berger, Jelinek, and Wolf [7], Jelinek [34], Jelinek and Schneider [35], Slepian and Wolf [53], Viterbi and Omura [57], Wolf [59], and Ziv [63]). Many others have reported results in this area [1,2,13,18,22,12,32]. Implementation of the various encoding techniques is generally not simple, although current large-scale integration technology is making implementation complex easier to handle. An exception is source encoding by quantization. For example, Berger [9] demonstrated an equivalence between perfectly entropy-encoded optimum quantizers and infinite block-length permutation codes, Chang [12] expanded and implemented Berger's results, and Morris [40] showed that the uniform quantizer generates an adequate rate-distortion bound (within 1 or 2 bits per second of Gaussian curve) for a class of sources that includes the Gaussian.

Classes of Sources

Published results on source encoding for a class of sources are few, and most of these results are recent. This problem has been treated by Berger [5,6], Davisson [15], Dobrushin [19,20], Morris [40], Neuhoff [41], Sakrison [47,48,50], Tan and Yao [55], and Ziv [63]. Berger used a two-person statistical game approach, with respect to a distortion measure, for a class of sources and a class of encoding schemes. He determined the exponential rate of growth (with block length) of the minimum number of code words needed to achieve a specified distortion level. Dobrushin gave an expression for the ϵ-entropy (a point on the rate-distortion function) of a class of sources. In both of these investigations, the class of sources were limited to discrete-time, discrete-amplitude sources. Morris theoretically verified the robustness (minimax) of the uniform quantizer for a class of discrete-time, memoryless sources with bounded amplitudes and then used this result to derive an upper bound on the rate-distortion function for that class of sources. This bound is achieved by the uniform quantizers. Sakrison, in two pioneering papers [47,48], extended Dobrushin's results to continuous-amplitude, discrete-time sources and to random processes. He stated that for a "compact" class of sources, the rate-distortion function for the class of sources is just the supremum of the rate-distortion functions of the individual sources and hence is more easily computed. In Ref. 50, Sakrison investigated robust encoding schemes and worst-case sources. He showed that for a class of ergodic sources satisfying a moment equality constraint, the Gaussian source is the worst-case source. He then conjectured that an optimum encoding for the worst-case source is the desired robust encoder. Tan and Yao presented a method of explicitly evaluating an absolute-magnitude-criterion, rate-distortion function for a class of independent and identically distributed sources having probability densities with constrained tail decay. Berger, Morris, Sakrison, and Tan, and Yao appear to be the only investigators making contributions in this area for the last 5 to 10 years.

Davisson [15], Neuhoff et al. [41], and Ziv [63], among others, have treated another form of source encoding, for a class of stationary sources, called universal coding. An encoder is called universal if the performance of the code, designed with only the knowledge that the true source is a member of a given class of sources, converges in the limit of long block length to the optimum performance possible if one knew the true source. Different types of universality, which correspond to different notions of convergence, are defined. Davisson considered universal noiseless coding; i.e., perfect reproduction of the source, where the performance measure is a function of the coding redundancy relative to the per-letter conditional source entropy. He also gives a short history of universal coding. Neuhoff et al. investigated universal coding with a fidelity criterion with the added constraint that the admissible codes have a given fixed code rate. The performance measure was some function of the distortion vs the minimum information rate. Ziv pioneered universal coding with a fidelity criterion [63, Part II]. He also considered universal coding for the performance measure, probability of coding error, using a fixed rate [63, Part I].

Implementation of these encoding techniques for classes of sources is generally difficult (e.g., Gilbert [23]), with the exception of quantization. In fact, for the problem formulations treated in Refs. 19, 20, 47, and 50 the encoding techniques for achieving

the rate-distortion function (ϵ-entropy in Refs. 19 and 20) performance, and the characterization of the worst-case source of the class, are not known. One exception seems to be for the class of sources with a given bound on the variances or with a given variance [47,50]. In these two cases, the Gaussian source is the worst-case source [51]. However, the form of the corresponding optimal encoding technique for the class of sources (i.e., the robust encoder) has only been conjectured [50]. For this reason, the robust quantization results of Morris [40], Morris and VandeLinde [95], and VandeLinde [111,112] have practical significance for memoryless sources.

Noise-Corrupted Sources

Source encoding of noise-corrupted source signals has had limited attention from rate-distortion theorists [44,49]. Assuming source-independent random noise and a mean-squared distortion measure, Sakrison [44] and Wolf and Ziv [49] formulated the problem in Hilbert space. Sakrison needed sufficiently long word lengths (i.e., large dimensionality) to take advantage of the assumed ergodicity of the source. Wolf and Ziv's results were applicable to any work lengths. They concluded [44,49] that the optimal encoding technique is first to compute the optimum conditional estimate of the source and then to encode this estimate optimally as if it were the source output. They derived expressions for the minimum possible distortion measure value, whose computational difficulty is generally greater than for the noiseless case. The problem of characterizing and implementing the optimal encoder still exists, since the source estimator is now the "source" to be optimally encoded.

Quantization

W. F. Sheppard, according to Bruce [70] and Smith [108], was the first to study a system of quantization. Sheppard, in 1898, essentially performed a round-off operation before making computations on the entries of a table of values [107]. The most referenced research work on quantization are probably the papers of J. Max [92] and B. Widrow [114]. Max considered the problem of the optimum N-level quantizer for the minimum mean weighted quantization error for a given signal. He also considered the optimum N-level uniform quantizer (i.e., equally spaced quantizer level values and equally spaced transition values). For the mean-squared error and sufficiently smooth signal density problem, Max showed that the centroids of the signal density area in each quantization interval are the optimum quantizer level values, and the midpoints between the optimum level values are the optimum transition points. The actual values must be computed recursively from the resulting necessary condition equations. Explicit results were given for the Gaussian source code. Widrow provided theoretical verification of the earlier experimental results of Bennett [66] by showing that the instantaneous quantization error, which is a signal-dependent error signal, can be considered statistically independent uniformly distributed noise when the number of quantization levels N satisfies a condition analagous to the Nyquist sampling theorem. He considered the uniform quantization operation as an area-sampling operation on the signal probability density function. When quantization frequency N is twice as high as the highest frequency component in the signal density function, the moments of the quantizer output signal are the same as the moments of the sum of the quantizer input signal and a statistically

independent noise uniformly distributed in a quantization interval. Satisfaction of this condition, according to his quantizing theorem, also allows complete recovery of the original probability density, given the quantizer output probability density.

Other Performance Measures

Most of the research on quantization schemes considered the mean weighted quantization error as the distortion measure, with the mean rth power quantization error the most common of that group. Other measures considered were based on special applications or information theoretic viewpoints that reflected the level of knowledge in the respective application areas. Signal-to-noise ratios were used as the distortion/performance measure for digitizing speech and picture signals in Refs. 73, 81, 82, 84, 102, and 108. The mean rth power of the quantization interval length, within which the signal is found, was used for determining bounds on quantizer performance Ref. 77. The quantizer output entropy was used as a basis for comparison of quantizer performance from an information theoretic viewpoint in Refs. 79, 80, and 94. An overall system performance measure may be used in some applications where the quantizer is considered just another part of a system. The overall system performance was the optimization goal in Refs. 68, 89, 90, and 110. Purton [102] and Smith [108] considered zero-memory prefiltering and post-filtering (called companding) of the quantizer input and output, to equalize S/N over the typical range of the speaker volume means. Their papers, especially Ref. 108, are probably the best treatments of companding. Cummiskey et al. [24], Gersho and Goodman [78], Golding and Schultheiss [81], Goodman and Gersho [82], and Jayant [87] investigated adaptive quantization to achieve the same equalization of S/N ratios. In this formulation, the interval size of the uniform quantizer is adjusted for the next signal value, which is predicted from knowledge of the previous values. Elias [77] derived lower bounds on his distortion measure for input signal density constraints of absolute continuity and finite domain. He also presented a good literature survey and an extensive bibliography. Messerschmitt [94] showed that the maximum-output-entropy quantizer and the minimum mean rth-power quantization-error quantizer are approximately the same, within a multiplicative constant, for a maximum-output-entropy goal. He required the source signal densities to be uniform or exponential in form. Goblick and Holsinger [80], in an important paper, used curves of the output entropy vs mean-squared quantization error as a basis for comparison of Gaussian source digitization schemes. Their often quoted result is that the uniform quantizer (Max's optimal quantizer) performance curve is 1/4 bit above the rate-distortion function (the theoretical limit) for the Gaussian source. Gish and Pierce [79] presented an asymptotic (in N) comparison between the uniform quantizer and the rate-distortion function for mean rth-power quantization-error distortion measure and reasonably smooth signal densities. They showed that the uniform quantizer yields an output entropy asymptotically smaller than that of any other quantizer, for reasonably-smooth signal density functions and a fairly general class of mean weighted quantization error distortion measures. Larson [89], Lewis and Tou [90], and Tou [110] used the system performance index or measure and considered the quantizer as just a small part of a dynamic sampled-data system. Bertram [68] obtained an expression for the upper bound of the error caused by quantization in such systems. Viswanathan and Marhoul [113] used a spectral sensitivity measure on a desired spectrum shape as the basis of quantizer optimization for application to digitally transmitted, linear predictive system parameters.

Optimal quantization with respect to the mean weighted-quantization-error distortion measure was investigated by many researchers after the paper by Max, the primary motivation being the interest in digital techniques, which began in the early 1960s. Bruce [69,70] developed a dynamic programing algorithm to specify the quantizer parameters that minimize this distortion measure (Ref. 70 has an extensive bibliography). The quantization-error weighting function in this formulation is not assumed to be symmetric or convex. However, unless the weighting function is convex, one must search for many possible relative extrema to obtain the optimum parameters. Roe [103] extended Max's results to mean rth-power quantization-error distortion measures and to signal density functions that are sufficiently differentiable to allow Taylor series expansions about the transition points. Algazi [64] and Wood [115] presented other approximation techniques for reducing the computational complexity and size of the quantization-parameter optimization algorithms given in previous papers [69,70,93,103]. The results of Algazi and Wood are based on large N and on sufficiently smooth signal density functions. Morris [40], Morris and VandeLinde [95], and VandeLinde [111,112] used a functional analysis formulation to verify the robustness of certain N-level quantizers in a minimax sense for the classes of sources.

Random Vector Quantization

Quantization of random vectors has been reported by only a few researchers. This type of formulation is especially important to information theorists because of their interest in encloding of signal source sequences (often called n-dimensional message, or block encoding), which is characteristic of multiplexed systems. Schutzenberger [107], in a relatively early paper, investigated the reciprocal relationship between the mean rth-power quantization error and the quantizer output entropy for the quantization of finite dimensional signals. He gave a lower bound to the distortion measure, which is proportional to $N^{-r/n}$. His results require continuous, bounded signal density functions that have a finite θ moment for some $\theta > r$ and have a finite entropy. The constant of proportionality was not explicitly presented. Huang [85] and Huang and Schultheiss [86] treated the quantization of random Gaussian vectors. They determined a matrix transformation that converted the correlated elements of each vector to uncorrelated elements and then quantized each element in an optimal manner. Zador [116], in a later, unpublished work, gave an asymptotic result that is similar to that of Ref. 105. Berger's [9] work on optimum quantizer and permutation code equivalence was formulated for random sequences. Segall [106] treated the problem of optimum allocation of the total number of available bits (quantization levels) among the components of a memoryless, stationary vector source. He gave an optimal decorrelating scheme for a source with dependent vector components.

Performance Experiments

Research concerned with the results of performance experiments on specific quantizers has been a very important field of quantization research. Bennett [66] was one of the first at this type of research. He demonstrated in his timely and important paper that the quantization-noise (error) power spectral density was uniformly distributed over the input signal band. He assumed a uniform quantizer with "more than a few number of intervals" and a Gaussian source with a flat band-limited power spectral density. After

analyzing his results, he suggested tapering the quantizer interval lengths so that the weak speech signals occurred in the shorter intervals and the strong speech signals in the longer intervals. This is the fundamental idea behind companding [102,108], as discussed earlier. O'Neal [99,100] investigated the performance of differential pulse code modulation and delta modulation systems for transmission of Gaussian and television signals. This method of encoding is often called predictive quantization, since only the quantized difference between the current signal value and the predicted value is transmitted. He also looked at the resulting quantization error. Damas et al. [76] and Thompson and Sparkes [109] experimentally investigated the effects of additive random noise, called dithering, on the signal for the purpose of whitening the quantization noise. This procedure seems to be very effective in digital picture processing. Arnstein [65] treated the quantization error in predictive coders for first-order Gaussian Markov sequences. His results indicate that the prediction error is close to Gaussian and that correlation of successive quantizer outputs does exists.

Sensitivity Analysis

The application of sensitivity analysis to quantizer optimization or performance evaluation has not been treated in the literature with any depth. A lack of an adequate sensitivity measure may have been the major inhibition to earlier use of sensitivity analysis. Viswanathan and Marhoul [115] were concerned with an accurate representation of the power spectrum for synthesizing good-quality speech. Thus, a spectral sensitivity measure was the basis for their quantizer optimization of the linear prediction parameter. Gray and Davisson [85] applied a generalization of the Vasershtein distance between source random variables to derive an upper bound on the difference in quantizer performance for different sources. Their bound provides a measure of the performance loss or mismatch that occurs in applying a quantizer to a source for which it was not designed.

Noise-Corrupted Sources

All previously referenced research treated the ideal case of noiseless input signals to the quantizer. Only a few investigators treated the quantization of noise-corrupted signal sources. In 1956, Myers [28] considered the uniform quantization of uniformly distributed sources with additive random Gaussian noise. He showed that for small-variance noise, the quantizer output would be essentially the same as the noiseless case, and an increased number of quantization intervals would yield good improvement in the total error. For large noise variances, the quantizer output would be incorrect a large percentage of the time, and an increased number of quantization intervals would yield very little improvement in the total error (i.e., the noise would dominate the total error). Myers suggested the probability of a correct reading as a measure of quantizer performance or as a measure of allowable noise limits. Myers appears to have been the first to have treated this type of problem. Bruce [70] extended this work and others by determining an algorithm for computing the optimum quantizer parameters that minimize the mean weighted-quantization-error distortion measure for noise-corrupted signal sources. The corrupting noise need not be additive. The other problem constraints are the same as his noiseless quantization problem. For the mean squared-error case, Bruce showed that the optimal N-level quantizer is identical to the optimal N-step approximation to the optimum mean

squared-error filter for the source. In general, the quantizer was viewed as merely an N-valued zero-memory filter. Kurtenbach and Wintz [88] determined optimum and optimum-uniform quantizer structures under the mean squared-error measure for the system that transmits digital data over a noisy channel. Their proposed design procedures required the channel transition probabilities to be related to the bit error rate. Gordan [83] very recently presented an engineering-oriented discussion on noise effects in analog-to-digital converters. This was similar to Myers' work. Morris [40,96] investigated the performance of an arbitrary quantizer for discrete-time, memoryless, noise-corrupted signal sources with a given amplitude bound. A functional analysis approach was used to derive the results on worst-case corrupting noise for arbitrary sources and quantizers. His results corroborated the findings of Myers.

Estimation and Control with Quantized Data

After these few results were obtained from the study of the optimal quantization of noise-corrupted signal sources, several researchers investigated the problem of signal estimation given the quantized data. This is a difficult problem because the conditional density functions are in general complex. Meier et al. [93] considered the effect of quantization of the linear measurement on state estimation of an unforced linear dynamic system. For linear estimation and certain assumptions on the density function at each time step, they presented a predictor-corrector filter of the Kalman filter form, for which the measurement noise covariance is effectively increased by an additive term representing the quantization noise. For nonlinear estimation and Gaussian assumptions, because the resulting conditional density functions would be non-Gaussian, approximate methods yielding tractable recursive estimates were presented. These approximate methods are based on the most probable state trajectory estimate or the most probable state and noise trajectory estimate. Curry et al. [74] and Curry [75] presented a general method of computing statistics, conditioned on quantized measurements, based on properties of the conditional expectation. On application to Gaussian discrete-time linear systems, a Kalman filter type of estimator was obtained. For nonlinear estimators, two approximation methods were used for obtaining tractable estimators. The first method uses a power series expansion of the Gaussian density function to approximate the mean and covariance of the conditional density function for a one-stage problem. The second method uses a Gaussian fit algorithm on the conditional density function at each stage, which yields a Kalman filter type of estimator. Clements [71] and Clements and Haddad [72] determined expressions for the conditional density functions from which approximate conditional mode and mean estimates can be determined. They treated nonlinear systems in this context. By using Taylor series expansions and making some assumptions on the system equations, predictor-corrector filter equations were obtained for the estimators. These equations reduced to the Kalman filter equations for linear systems and no quantization.

References 71 and 72 are more general than the others because they treat the state estimation of fairly general nonlinear systems. However, Curry's book [75] appears to be the only one devoted entirely to estimation and control of systems with quantized measurements. It also contains an extensive bibliography.

SUMMARY AND CONCLUSION

Significant research results relating to encoding information sources have been highlighted in this survey. Particular attention has been given to the application of quantization and its importance in source encoding in a fidelity criterion context.

Since source encoding with a fidelity criterion (i.e., rate-distortion theory) is a relatively new area of research activity, the literature is largely theoretical. Most research has been confined to determining source encoding schemes and rate-distortion functions for independent, stationary, Gaussian sources with respect to mean weighted-squared-error distortion measures. Some theoretical results have been obtained also for certain deviations from this model, most notably for the minimum rates of Gaussian sources without the independence or ergodic assumptions and for the performance of specific coding or quantization schemes with respect to the standard models. Although the earliest results were generally lacking in practicality or implementability, with the exception of quantization, they did provide insight and an understanding of the problem.

There are still problems that are unsolved or in early stages of development. These include development of adaptive and/or robust practical source encoders for classes of general sources, sources with memory, nonstationary sources, and sources with vector outputs; development of adaptive and/or robust quantizers for these same types of sources; application of more subjective distortion measures or fidelity criteria for sources with memory and symbol context dependencies, such as in speech or visual images; and determining the rate-distortion functions for these sources and distortion measures. Successful investigation of these areas would considerably advance the use of this theory in telecommunication systems design.

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